

TRANSIENT HEAT AND MASS TRANSFER FLOW THROUGH SALT WATER IN AN OCEAN WITH HEAT GENERATION

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ABSTRACT

The coastal ocean is a region receiving a great deal of attention due to an increasing utilization of its resources. Fluids move and accelerate under the action of forces like gravity and surface-contact forces. The prognostic variables are mainly velocity, temperature and salinity. The velocity, temperature, and salinity equations are nonlinear. The numerical solutions for the velocity field, temperature distribution, salinity distribution are obtained by the explicit finite difference method. Also the stability and convergence criteria of the model are established. The obtained results have been shown graphically for salt water in an ocean with heat generation.

Keywords: Prandtl Number, Heat Source Parameter, Salinity.

1. INTRODUCTION

Ocean covers 70% of the earth surface. It offers tremendous either biological or mineral resources as well as the possibilities for the navigation and resort for human beings. It is of great importance to develop and utilize the ocean resources with the concepts of sustainable development. In order to improve the sustainable development of ocean resources, the well understanding and acknowledgement of oceanographic and meteorological phenomena should be emphasized.

The ocean is now well known to play a dominant role in the climate system, because it can initiate and amplify climate change on many different time scales. The best known example is inter-annual variability of El Niño [1] and the potential modification of the major patterns for oceanic heat transport as a result of increasing greenhouse gases [2]. Yet the ocean has been very much under measured for most of the history of Ocean Science. Even though systematic observations began in the 1880s with pioneering observations by Nansen et.al [3], the seagoing and theoretical efforts were mainly oriented toward describing large scale circulation [4], which was often regarded as steady for lack of more detailed information. To gain some appreciation for the model's ability to simulate coastal Ocean circulation Blumberg and Mellor [5, 6, 7, 8, 9, and 10] has investigated the potential impact, mathematical modeling, and physical behavior for their model. These applications include a simulation of the tides in

the Chesapeake Bay, a simulation of the coastal circulation, off long Island, New-York, and a computation of the general circulation in the Middle-Atlantic and South-Atlantic Bights and in the Gulf of Mexico.

Ezer [11] studied the importance of Ocean circulation model. He has created and maintain POM web site. Institutionally the model was developed and applied to Oceanographic problems, the Geophysical Fluid Dynamics, Laboratory of NOAA and Dynalysis of Princeton.

Hence our aim is to study explicit finite difference solution of transient heat and mass transfer flow through salt water in an ocean with heat generation. Explicit finite difference method has been used to solve the problem with different time step.

2. MATHEMATICAL MODEL OF FLOW

Introducing the Cartesian co-ordinate system the x -axis is chosen along the plate in the direction of flow and the y -axis is normal to it. Initially we consider that the plate as well as the fluid is at the same temperature $T(T_\infty)$ and the Salinity level $S(S_\infty)$ everywhere in the fluid is same. Also it is assumed that the fluid and the plate is at rest after the plate is to be moving with a constant velocity. In this case the plat has been considered vertical.

U_0 is its own plane and instantaneously at time $t > 0$, the temperature of the plate and spices salinity

raised to $T_w (> T_\infty)$ and $S_w (> S_\infty)$ respectively, which are these after maintained constant where T_w, S_w are temperature and spices salinity at the wall and T_∞, S_∞ are the temperature and salinity of the spices far away from the plate respectively. The physical model is furnished in the figure1.

Under the boundary layer approximation the governing equation for the transient heat and mass transfer through salt water in an ocean with heat generation are given below

The continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

The momentum equation

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(S - S_\infty) \quad (2)$$

The Salinity equation

$$\frac{\partial s}{\partial t} + u \frac{\partial s}{\partial x} + v \frac{\partial s}{\partial y} = K_s \frac{\partial^2 s}{\partial y^2} + F_s \quad (3)$$

The temperature equation

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = K_T \frac{\partial^2 T}{\partial y^2} + \frac{Q_T}{\rho C_p} \quad (4)$$

The initial and boundary conditions are

$$\begin{aligned} t \leq 0, u = 0, v = 0, T \rightarrow T_\infty, S \rightarrow S_\infty \quad \text{everywhere} \\ t > 0, u = 0, v = 0, T \rightarrow T_\infty, S \rightarrow S_\infty \quad \text{at } x = 0 \\ u = 0, v = 0, T \rightarrow T_\infty, S \rightarrow S_\infty \quad \text{at } y = 0 \\ u = 0, v = 0, T \rightarrow T_\infty, S \rightarrow S_\infty \quad \text{as } y = \infty \end{aligned} \quad (5)$$

where ν is the kinematic viscosity, K_s is the thermal conductivity for salinity, K_T is the thermal conductivity for temperature, F_s is the molecular diffusion, C_p is the specific heat at constant pressure, U is the uniform velocity.

3. MATHEMATICAL FORMULATION

Since the solution of the governing equations under the initial and boundary conditions will be based on an explicit finite difference method. It is required to make the equations dimensionless. For this purpose, we introducing the following dimensionless variables in equations (1) to (5) and get the following equations (6) to (11) and the initial and boundary conditions;

$$X = \frac{xU_0}{\nu}, Y = \frac{yU_0}{\nu},$$

$$U = \frac{u}{U_0}, V = \frac{v}{U_0},$$

$$\tau = \frac{tU_0^2}{\nu}, \bar{T} = \frac{T - T_\infty}{T_w - T_\infty}$$

$$\text{and } \bar{S} = \frac{S - S_\infty}{S_w - S_\infty}, Q_T = (T - T_\infty)Q^*$$

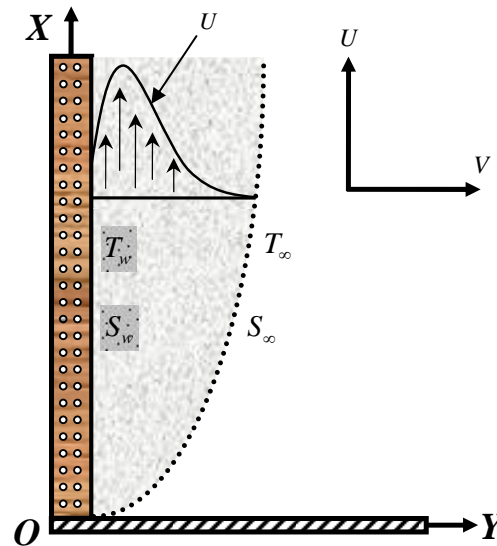


Fig 1. Physical model and coordinate system

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (6)$$

$$\frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = G_r \bar{T} + G_s \bar{S} + \frac{\partial^2 U}{\partial Y^2} \quad (7)$$

$$\frac{\partial \bar{S}}{\partial \tau} + U \frac{\partial \bar{S}}{\partial X} + V \frac{\partial \bar{S}}{\partial Y} = \frac{2}{P_s} \frac{\partial^2 \bar{S}}{\partial Y^2} \quad (8)$$

$$\frac{\partial \bar{T}}{\partial \tau} + U \frac{\partial \bar{T}}{\partial X} + V \frac{\partial \bar{T}}{\partial Y} = \frac{1}{P_r} \frac{\partial^2 \bar{T}}{\partial Y^2} + \bar{T}\alpha \quad (9)$$

And the non-dimensional boundary conditions are;

$$\tau \leq 0, U = 0, V = 0, \bar{T} = 0, \bar{S} = 0 \quad \text{everywhere} \quad (10)$$

$$\tau > 0, U = 0, V = 0, \bar{T} = 0, \bar{S} = 0 \quad \text{at } X = 0$$

$$U = 0, V = 0, \bar{T} = 1, \bar{S} = 1 \quad \text{at } Y = 0 \quad (11)$$

$$U = 0, V = 0, \bar{T} = 0, \bar{S} = 0 \quad \text{as } Y = \infty$$

where Prandtl number $P_r = \frac{\rho \nu C_p}{K}$; Modified Prandtl

number $P_s = \frac{\rho \nu C_s}{K}$, Grashof number

$$G_r = \frac{g\beta \nu (T - T_\infty)}{U_0^3}, \text{ Modified Grashof number}$$

$$G_s = \frac{g\beta^* \nu (S - S_\infty)}{U_0^3}, \text{ Heat source parameter}$$

$$\alpha = \frac{Q^* \nu}{U_0^2 \rho C_p}$$

4. NUMERICAL SOLUTIONS

In order to solve the non-dimensional system by the explicit finite difference method, it is required a set of finite difference equations. For this, a rectangular region

of the flow field is chosen and the region is divided into a grid of lines parallel to X and Y axes, where X -axis is taken along the plate and Y -axis is normal to the plate. Here we consider that the plate of height $X_{\max} (= 25)$ i.e. X varies from 0 to 25 and assumed $Y_{\max} (= 25)$ as corresponding to $Y \rightarrow \infty$ i.e. Y varies from 0 to 25. There are $m (= 250)$ and $n (= 250)$ grid spacing in the X and Y directions respectively as shown in figure 2. It is assumed that ΔX , ΔY are constant mesh sizes along X and Y directions respectively and taken as follows, $\Delta X = 0.1 (0 \leq X \leq 25)$ and $\Delta Y = 0.1 (0 \leq Y \leq 25)$ with the smaller time-step, $\Delta \tau = 0.001$.

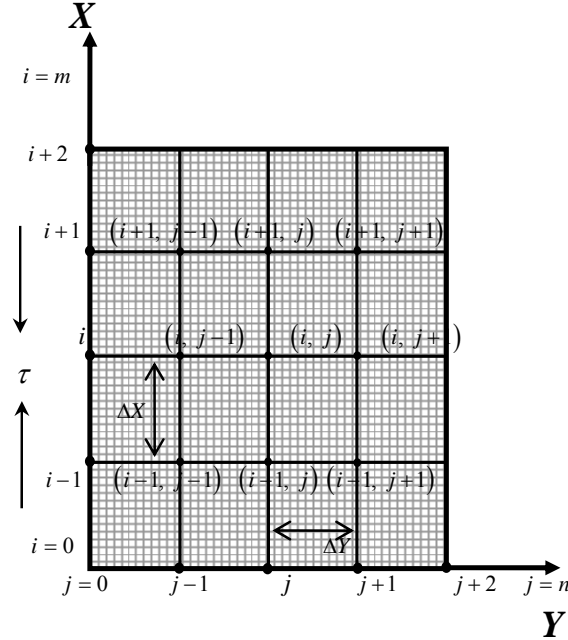


Fig 2. Finite difference space grid.

Let U' , V' , \bar{T}' and \bar{S}' denote the values of U , V , \bar{T} and \bar{S} at the end of a time-step respectively. Using the explicit finite difference approximation, we obtain the following appropriate set of finite difference equations;

$$\frac{U'_{i,j} - U'_{i-1,j}}{\Delta X} + \frac{V_{i,j} - V_{i,j-1}}{\Delta Y} = 0 \quad (12)$$

$$\frac{U'_{i,j} - U_{i,j}}{\Delta \tau} + U_{i,j} \frac{U_{i,j} - U_{i-1,j}}{\Delta X} + V_{i,j} \frac{U_{i,j+1} - U_{i,j}}{\Delta Y} = G_r \bar{T} + G_s \bar{S} + \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(\Delta Y)^2} \quad (13)$$

$$\frac{\bar{S}'_{i,j} - \bar{S}_{i,j}}{\Delta \tau} + U_{i,j} \frac{\bar{S}_{i,j} - \bar{S}_{i-1,j}}{\Delta X} + V_{i,j} \frac{\bar{S}_{i,j+1} - \bar{S}_{i,j}}{\Delta Y} = \frac{2 \bar{S}_{i,j+1} - 2\bar{S}_{i,j} + \bar{S}_{i,j-1}}{P_s (\Delta Y)^2} \quad (14)$$

$$\frac{\bar{T}'_{i,j} - \bar{T}_{i,j}}{\Delta \tau} + U_{i,j} \frac{\bar{T}_{i,j} - \bar{T}_{i-1,j}}{\Delta X} + V_{i,j} \frac{\bar{T}_{i,j+1} - \bar{T}_{i,j}}{\Delta Y} =$$

$$\frac{1}{P_r} \frac{\bar{T}_{i,j+1} - 2\bar{T}_{i,j} + \bar{T}_{i,j-1}}{(\Delta Y)^2} + \bar{T}_{i,j} \alpha \quad (15)$$

with initial and boundary conditions

$$U_{i,j}^0 = 0, V_{i,j}^0 = 0, \bar{T}_{i,j}^0 = 0, \bar{S}_{i,j}^0 = 0 \quad (16)$$

$$U_{0,j}^n = 0, V_{0,j}^n = 0, \bar{T}_{0,j}^n = 0, \bar{S}_{0,j}^n = 0$$

$$U_{i,0}^n = 1, V_{i,0}^n = 0, \bar{T}_{i,0}^n = 1, \bar{S}_{i,0}^n = 1$$

$$U_{i,L}^n = 0, V_{i,L}^n = 0, \bar{T}_{i,L}^n = 0, \bar{S}_{i,L}^n = 0, \quad L \rightarrow \infty$$

Here the subscripts i and j designate the grid points with X and Y coordinates respectively and the superscript n represents a value of time, $\tau = n \Delta \tau$ where $n = 0, 1, 2, \dots$. The stability conditions of the method are

$$U \frac{\Delta \tau}{\Delta X} + |V| \frac{\Delta \tau}{\Delta Y} + \frac{1}{P_r} \frac{\Delta \tau}{(\Delta Y)^2} \leq 1 + \frac{\alpha}{2},$$

$$U \frac{\Delta \tau}{\Delta X} + |V| \frac{\Delta \tau}{\Delta Y} + \frac{2}{P_s} \frac{\Delta \tau}{(\Delta Y)^2} \leq 1 \text{ and convergence criteria}$$

of the method are $P_r \geq 0.08$ and $P_s \geq 0.20$. And the fixed value of heat source parameter $\alpha = 0.5$.

5. RESULTS AND DISCUSSION

In order to discuss the physical situation of the model we have computed the numerical values of the non-dimensional velocity (U), temperature (T) and salinity (S) with the boundary layer for Prandtl number (P_r), and modified Prandtl number (P_s) for salinity, with the arbitrary values of Grashof number (G_r), modified Grashof number (G_s) and heat source parameter (α).

To get the steady-state solution, the computations have been carried out up to dimensionless time $\tau = 80$ the result of the computations show little changes in the above mentioned quantities after dimensionless time $\tau = 60$ have been reached. Thus the solutions of the variables for $\tau = 80$ are essentially steady-state. Hence the velocity, salinity, temperature profiles are drawn for $\tau = 10$ and $\tau = 80$.

The effect of Changing Prandtl number is shown in the Figs. 3-5. Here the value of Prandtl number is taken 0.50, 0.71, 1.0, 7.0. And the fixed value of modified Prandtl number ($P_s = 1.0$). The effect of modified Prandtl number (P_s) shown in the figs. 6-8. The value of the modified Prandtl number (P_s) is taken at the different salinity percentage in the ocean. For 15% salinity the value of (P_s) is 8.234, 25% salinity the value of (P_s) is 10.234, 30% salinity the value of (P_s) is 11.234, 35% salinity the value of (P_s) is 13.234. For the value of modified Prandtl number (P_s) we use the fixed value $P_r = 7.0$, $G_r = 1.50$, $G_s = 1.0$, $\alpha = 0.50$.

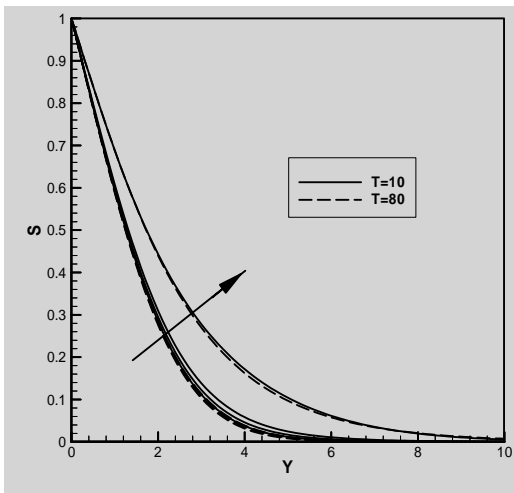


Fig 3. Salinity profile for different values of P_r at time $\tau = 10, 80$ with $P_s = 1.0, G_r = 1.50, G_s = 1.0, \alpha = 0.50$

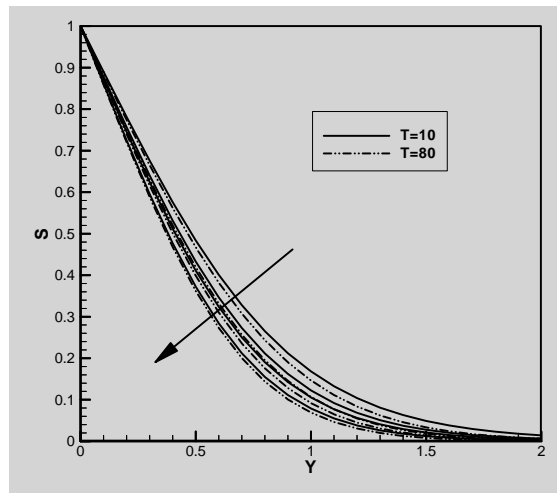


Fig 6. Salinity profile for different values of P_s at time $\tau = 10, 80$ with $P_r = 7.0, G_r = 1.50, G_s = 1.0, \alpha = 0.50$

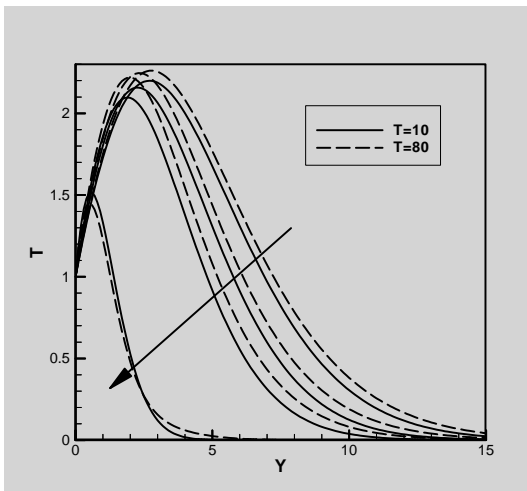


Fig 4. Temperature profile for different values of P_r at time $\tau = 10, 80$ with $P_s = 1.0, G_r = 1.50, G_s = 1.0, \alpha = 0.50$

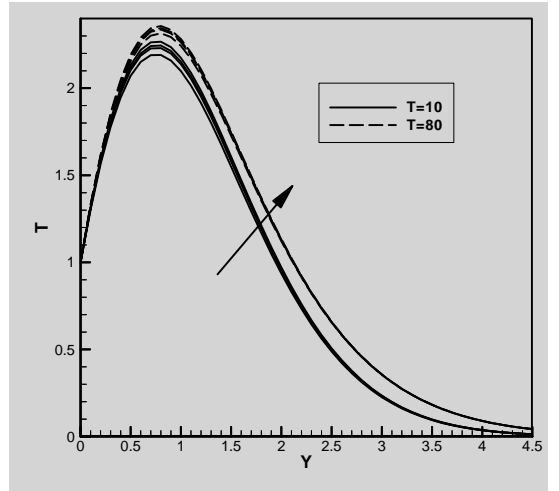


Fig 7. Temperature profile for different values of P_s at time $\tau = 10, 80$ with $P_r = 7.0, G_r = 1.50, G_s = 1.0, \alpha = 0.50$

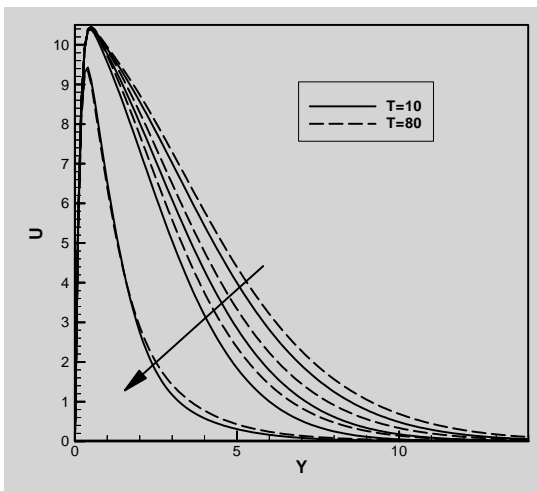


Fig 5. Velocity profile for different values of P_r at time $\tau = 10, 80$ with $P_s = 1.0, G_r = 1.50, G_s = 1.0, \alpha = 0.50$

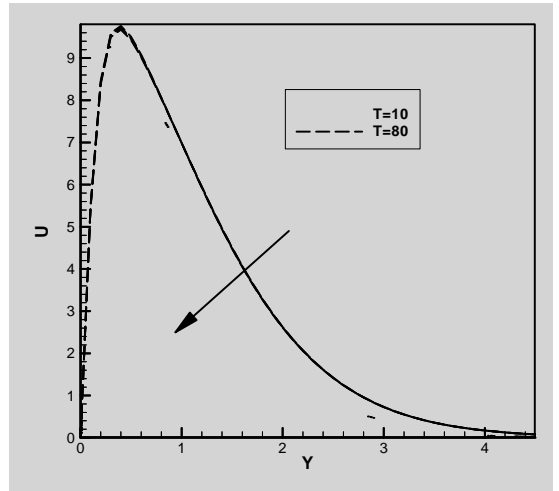


Fig 8. Velocity profile for different values of P_s at time $\tau = 10, 80$ with $P_r = 7.0, G_r = 1.50, G_s = 1.0, \alpha = 0.50$

6. CONCLUSION

It is observed that, for different values of Prandtl number

- Salinity distribution increases as P_r increase.
- Temperature distribution decreases as P_r increase.
- Velocity distribution decreases as P_r increase.
And for different values of Modified Prandtl number
- Salinity distribution decreases as P_s increase.
- Temperature distribution increases as P_s increase.
- Velocity distribution decreases as P_s increase.

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8. NOMENCLATURE

Symbol	Meaning	Unit
x, y	Cartesian coordinates	
u, v	Velocity components	(ms^{-1})
ν	Kinematic viscosity	(m^2s^{-1})
τ	Dimensionless time	
X, Y	Dimensionless cartesian coordinates	
U, V	Dimensionless velocity components	
\bar{T}	Dimensionless temperature	
\bar{S}	Dimensionless Salinity	
G_r	Grashof number	
G_m	Modified Grashof number	
P_s	Modified Prandtl number	
P_r	Prandtl number	
G_s	Modified Grashof number	

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